Research on Multi-Objective Optimization Based on Computable General Equilibrium Policy Analysis Model

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Abstract: Based on the existing Computable General Equilibrium (CGE) policy analysis model, a set of Computable Food Equilibrium (CFE) system models are established. Efficiency, profitability, sustainability, and equity are introduced as essential indicator of our model. To optimize and predict the data selected for sustainability and equity, factor analysis is used to select the main relevant indicators. According to the existing multi-objective programming model, function programming is applied to the optimization of sustainability. By applying neural network model, the time of implementation of optimized system is predicted, which is 2027. Through the simulation of the CFE model, we have found that the model is particularly approximate for data from developing countries, and in the case study of Mexico we found that our system has some advantages to investigate indicators in terms of equity and sustainability.

1. Background

The recent series of food crises have shown that even in some developed regions of the world, our global food system is still not stable enough. And our current global system of massive national and international food producers and distributers that allows relatively cheap and efficient food production and distribution is a certain cause of this instability. Therefore, food system is attached increasing importance contemporarily. Although the food produced in the world is theoretically enough to feed everyone in the world, under the current food system, people who cannot afford enough nutritious food are still spread across every continent, even if they are in rich countries. Considering some long-term factor, like continual growth in global population and decreasing resources, how to maintain or even improve economic status while producing more food has become an important issue worthy of our efforts. A CFE system that prioritizes equity and sustainability is proposed to mitigate some above negative influences.

In this paper, 3 primary foods, cereal, vegetable and meat are selected based on factor analysis method, which lay foundation on our following analysis. Equity and sustainability are studied using a multi-objective programming model and BP-Neural Network. CFE model is used to simulate and evaluate the food systems in Mexico, specifically focusing on profit and incidence of undernourishment.

2. Model Establishment

2.1 parameter settings

Efficiency:When discussing food production, we focus on the discussion of the aggregate price of value added (***PVA***) and the price of the ***ND*** bundle (***PND***). Here, we give the production function [1]:

$$PX = A[\delta_1 PND^{-\rho} + \delta_2 PVA^{-\rho}]^{-\frac{1}{\rho}} \tag{1}$$

Profitability:When discussing food trade, we first carry out Armington estimation and get the Armington price. Then estimate the food price based on the food price index and the world price of exported food, as follows:

$$PA = [\alpha_1 PD^{-\mu} + \alpha_2 PM^{-\mu}]^{-\frac{1}{\mu}} \tag{2}$$

Equity:To ensure the equity of food supply in different countries and regions, it is necessary to balance supply and demand in the domestic and foreign markets. Therefore, we can obtain the following relationship:

$$XD^d = XD^s \tag{3}$$

$$ED = ES \tag{4}$$

Among them,$\boldsymbol{XD^d}$ represents the domestically produced good, and XD^s represents the domestic market. With XP representing aggregate output, the component supplied to the domestic market is XD_s ,ED stands for export demand, ES stands for the component allocated to foreign markets [2].

Sustainability:Residents' income and demand for food are important factors to ensure the sustainability of food supply. We use the following two relations to describe the sustainability of food supply: When $XAC > \theta$:

$$XAC = \theta + \alpha \frac{Y}{PA} \tag{5}$$

Where $\boldsymbol{XAC}$ is the consumer demand for the Armington good, θ is the the so-called subsistence minima, α is the economic coefficient of excess consumption, and γ is the total amount of excess consumption. When $XAC < \theta$ represents the undernourishment rate, n_i and n_j represent the undernourished people and the total number of people, respectively [3]. The food structure diagram is shown in Figure 1.

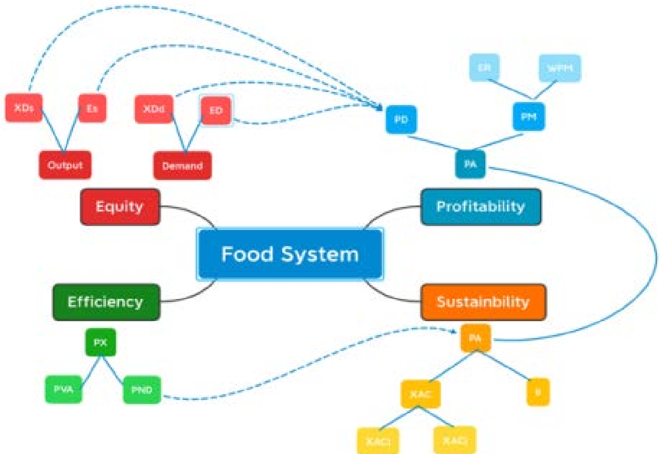


Figure 1. The Food Structure Diagram

2.2 Application of Factor Analysis Model

When quantitatively assessing the equity and sustainability of the food system, we will establish a model and use factor analysis method to evaluate the equity and sustainability of the food system more comprehensively.

Eight factors that are highly related to equity and sustainability are selected, which are "Cereal", "Vegetable", "Meat", "Milk", "Fruit", "Condiment", "Aquatic product", and "Special nutrition food".

Factor analysis is applied to analyze the impact of the eight factors on equity and sustainability in the food system. Suppose there are two variables of "priority" and "sustainability",$m = 2$, mark $Z_1 =$ "$priority$", $Z_2 =$ "sustainbility" . Common factor $F_1 =$ "$cereal$" , $F_2 =$ "$vegetable$", $F_3 =$

"$Meat$", $F_4 =$ "$milk$", $F_5 =$ "$fruit$", $F_6 =$ "$condiment$", $F_7 =$ "$Aquatic\ product$", $F_8 =$ "$Special\ nutrition\ food$", the linear equation can be obtained from this:

$$\begin{cases} Z_1 = a_{11}F_1 + a_{12}F_2 + a_{13}F_3 + \cdots + a_{18}F_8 \\ Z_2 = a_{21}F_1 + a_{22}F_2 + a_{23}F_3 + \cdots + a_{28}F_8 \end{cases} \tag{6}$$

r_{ij} is the correlation of Z_i and Z_j, The sample correlation matrix of Z is an $m-order$ symmetric matrix, that is $R_{ij} = (r_{ij}) = \begin{pmatrix} 1 & r_{12} & r_{13} & \cdots & r_{1m} \\ r_{21} & 1 & r_{23} & \cdots & r_{2m} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ r_{m1} & r_{m2} & r_{m3} & \cdots & 1 \end{pmatrix}$.

According to the stipulated $N = 8$ common factors, the cumulative variance contribution rate reaches 85%, so that N satisfies $\frac{\sum_{i=1}^{N} n}{\sum_{1}^{m} m} \geq 0.85$ is the number of common factors taken.

Table 1 shows contribution rate for eight factors and cumulative contribution rate. It can be concluded that the functional relationship of 8 factors is:

$$Y = 0.5126F_1 + 0.2172F_2 + 0.1253F_3 + 0.0456F_4 + 0.0352F_5 + 0.0295F_6 + 0.0194F_7 + 0.0162F_8$$

Among them, F1="cereal", F2= "vegetable", F3= "meat", F4= "milk", F5= "fruit", F6= "condiment", F7= "Aquatic product", F8="Special nutrition food".

Table 1. Factor Analysis

Factor	Contribution Rate	Cumulative Contribution Rate
Cereal	51.26%	51.26%
Vegetable	21.72%	72.98%
Meat	12.43%	85.41%
Milk	4.56%	89.97%
Fruit	3.52%	93.49%
Condiment	2.95%	96.44%
Aquatic product	1.94%	98.38%
Special nutrition food	1.62%	100.00%

2.3 Sustainability Programming

Reasonable pricing of the minimum purchase price is an important guarantee for sustainable development. Multi-objective programming model, specifically variable programming, are used to determine the minimum purchase price of cereal, vegetable, meat. In this model, it is assumed that food production will increase, and market prices will stabilize during 2016 to 2020.

Establishment of objective function:

$$\begin{cases} f_1(x) = \sum_{i=1}^{n} L_i x_1 + b_i \\ f_2(x) = \sum_{j=1}^{n} L_j x_2 + b_j \\ f_3(x) = \sum_{k=1}^{n} L_k x_3 + b_k \end{cases} \tag{7}$$

$f_1(x)$is the cereal output function, where L_i represents the grain output of the given $i-th$ year. b_i is the disturbance of cereal production. $f_2(x)$ is the yield output function, where L_j represents the yield of vegetables in the given year j. b_j is the disturbance of vegetable production. $f_3(x)$ is the function of meat, where L_k represents the yield of meat in the given year k. b_k is the disturbance of meat production.

Establish Constraints: The minimum purchase price that is too high or too low will have an adverse effect on market prices, grain production, and national finances. Therefore, constraints such as state subsidies, cereal storage, consuming capability, spare production of vegetable are added to this model. Obtain the following constraints:

$$(x_a - m_i)L_i \leq \sigma_a m_{i-1} L_{i-1} \tag{8}$$

Among them, the first constraint indicates a restriction on the amount of state subsidies. $x_a(a = 1,2,3)$ represents the lowest prices of grains, vegetables, and meat, respectively, and m_i represents the corresponding $i-th$ market price. L_i represents the output of the corresponding $i-th$ year. $\sigma_a = (a = 1,2,3)$represents the corresponding subsidy limits for grains, vegetables, and meat, and take 0.1, 0.08, and 0.07 respectively.

$$(x_1 - m_i)L_j \geq \beta_1(x_1 - m_{i-1})L_{j-1} \tag{9}$$

To achieve sustainability, storage coefficient should be considered because it is significant to store cereal in case natural disaster occur. Betta1 represents storage coefficient:

$$\frac{x_2 - m_{j-1}}{m_{j-1}} \leq \beta_2 \frac{S_2 - S_{j-1}}{S_j - 1} \tag{10}$$

If meat price increase, consumers burden will increase at the same time. The increase of the minimum purchase price should be less than the increase of residents ’income by 105%, which is demoted as β_2[5].

$$(x_3 - M_k)L_k \geq (x_3 - M_{k-1})L_{k-1} \times q \tag{11}$$

Spare production coefficient should be considered because vegetable may rot Spare production coefficient should be considered because vegetable may rot in the short period of time. q represents spare production coefficient.

$$x_{L_a} \leq x_a \leq x_{m_a} \tag{12}$$

The minimum purchase price has a limited range, which should between the minimum and maximum of market price.

Table 2. Minimum Purchase Price

Food	Price Range	2016	2017	2018	2019	2020
Cereal	4.39-5.08	4.54	4.47	4.42	4.52	4.43
Vegetable	1.93-2.24	2.01	1.95	1.97	2.02	1.94
Meat	37.44-39.53	37.76	37.62	37.54	37.47	37.56

As shown in the table 2, the minimum purchase prices of 3 food from 2016 to 2020 are obtained. When the minimum purchase price is closer to the lower bound of price range, food system is more sustainable financially. The majority of the above data are less than the minimum purchase price that we collected in the reality mostly, indicating that it has practical significance. By applying the optimal minimum purchase price, it can increase the production of food and release the burden of consumption. Under that circumstance, the food system can develop sustainably. By applying multi-objective programming model, equity and sustainability are greatly prioritized in comparison to current food system.

2.4 BP-Neural Network

The neural network model is an algorithmic mathematical model based on sufficient available data that is related, which is a widely accepted model to apply in prediction.

As a multi-layer feedforward neural network, BP neural network is composed of input layer, hidden layer and output layer. The most important stage is that when the output layer cannot get the expected output, it will switch to back propagation, and adjust the network weight and threshold according to the prediction error. [7]

In this model, Bayesian-regularization training method is applied, and we use the national grain yield per hectare and per capita grain area to reflect and predict the two indexes of "sustainability" and "equity" respectively.

The program structure diagram is as follows:

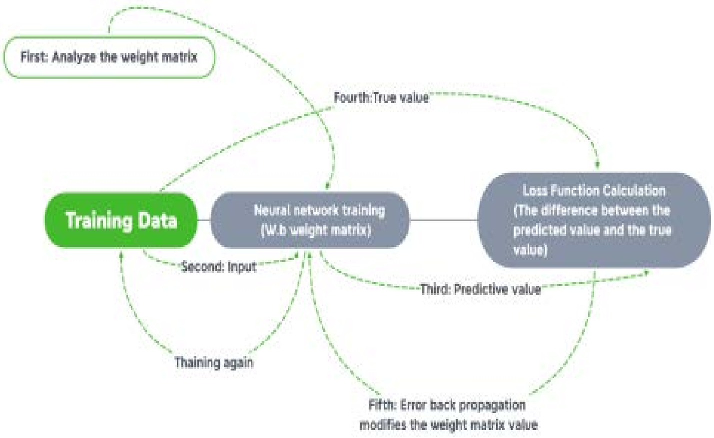


Figure 2. The structure Diagram

3. Result Output and Prediction

As shown in the figure 3, left figure represents mean square error of per capita cultivated land and right figure represents mean square error of total food production. Both mean square errors are relatively low, indicating that the results of prediction are accurate enough to predict the implementation of this food system.

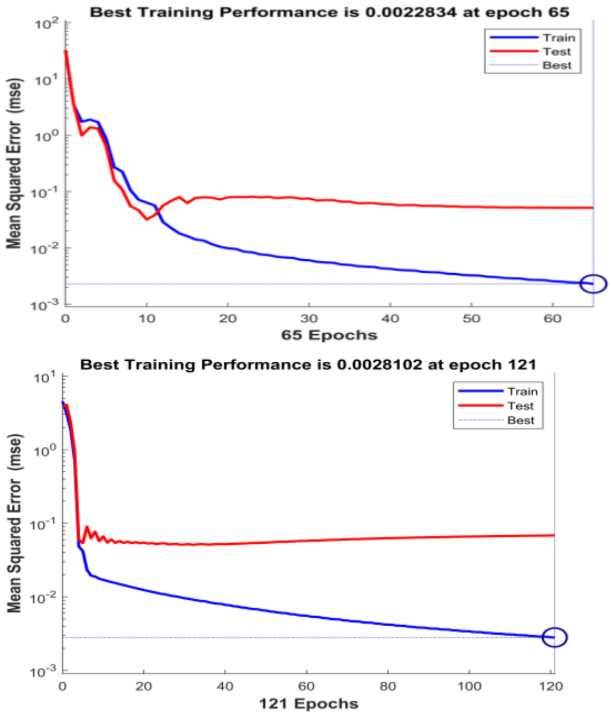


Figure 3. Result

Year	Total Production	Equity Rate of Per Capita Cultivated Land
2021	7895	62.04%
2022	8142	64.54%
2023	8402	66.52%
2024	8714	68.84%
2025	9048	71.43%
2026	9413	73.56%
2027	9824	75.98%

Figure 4. Prediction

Applying neural network model, we get the result of prediction. The table illustrates that the total production (in kilotons) and equity rate of per capita cultivated land. The requirements of optimized food system are 9000 kilotons for total production and 75% for equity rate, respectively. According to statistics, total production meets its requirement in 2025 and per capita cultivated land meets its requirement in 2027. Accordingly, in 2027, this food system can achieve optimal equity and sustainability. At that time, sufficient production can ensure citizen have enough food and it is also beneficial for economic growth, promoting per capita grain possession.

4. Simulation

It is acknowledged that the current food system has some obvious defects, which requires some positive change. In our CFE model, the equity and sustainability are prioritized.

(a) Developed Countries (b) Developing Countries

Figure 5. Price Index

According to Figure 5, the price index for developing countries has increased from 2014 to 2018, showing an increasing trend. Applying equation (3) and the positive correlation between the final unit cost and the price index, the price index can be obtained. A lower price index indicates a lower cost for the CFS system and an increase in the price index indicates that the food system requires more costs. Figure 5 shows the total consumer profit in developing countries. For the last two decades. Overall, the growth rate of total consumption profit has continued to increase. In Figure, the two curves largely coincide over the cycle, which means that our simulated values are quite appropriate.

Through the data we obtained from the Statistical Yearbook, we have introduced the price indices of Agricultural Products, Capital Goods, Industrial Products, Raw material, Consumers' Goods in Mexico. Based on the CFE system we established, we use formula (1) to simulate the cost of our model. By analyzing the price index of cost, the efficiency of the food system can be well reflected because production costs have a great relationship with labor efficiency.

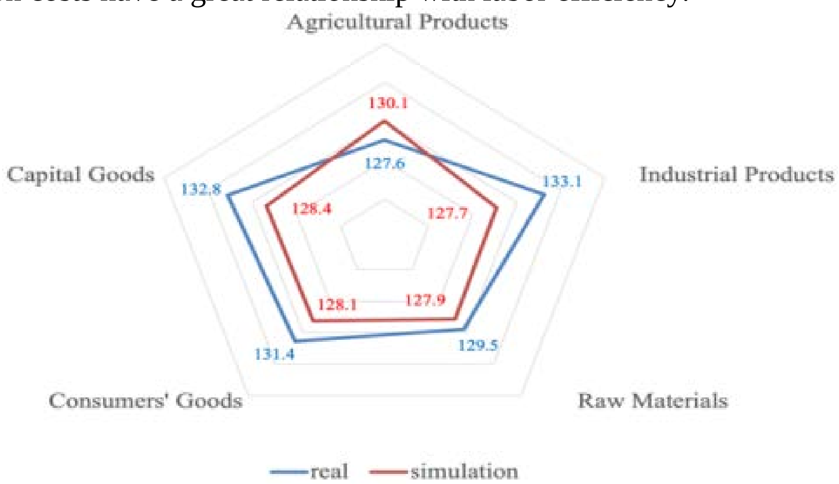


Figure 6. Price Index in 5 Main Categories

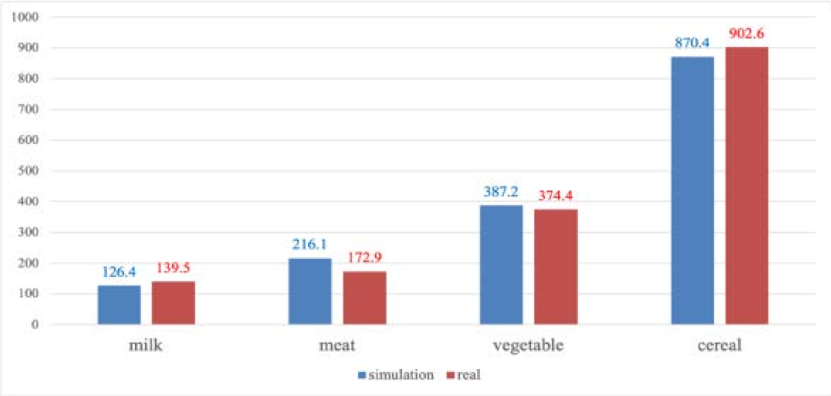
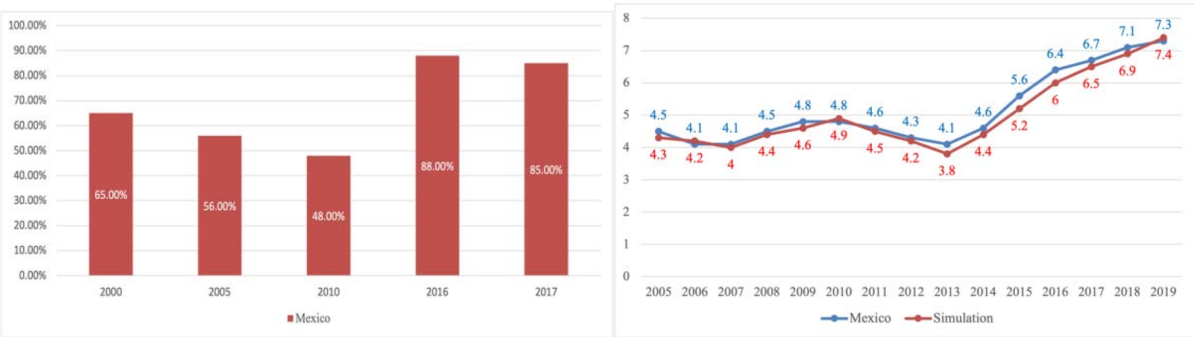


Figure 7. Price Index in 5 Main Categories



(a) Equity Improvement (b) Incidence of Undernourishment

Figure 8. Equity

Although we have analyzed 5 aspects, the most relevant to food is the price index of agriculture products. As shown in Figure 7, the simulation data of meat and vegetable in Mexico is increasing, and the other two are decreasing. Considering the relative factors to measure equity, we adopt the formula (4), (5) of the CFE system. Through the statistical yearbook import and export consumer food prices, we analyze the simulated consumer prices of the CFE system and compare them with the real consumer prices to calculate a ratio like the degree of equity improvement.

As shown in Figure 8(a), we modelled the equilibrium growth rate through equation 6, with a significant improvement in Mexico, in terms of sustainability, we calculated the incidence of malnutrition through formula 5 and table1 in the CFE system. The prevalence of malnutrition has important implications for the sustainability of the region. As shown in Figure 9(b), the modelled data for each year in Mexico show some degree of decline compared to the real data, although an increase in individual years cannot be excluded.

5. Conclusion

To sum up, CFE model uses quantitative formulas for efficiency, profitability, sustainability and equity to carry out subsequent optimization, prediction, comparison, and evaluation processes. We find that the CFE model is effective and stable and is of great help to developing countries. Because our model does not consider uncontrollable factors, more future research is needed.

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